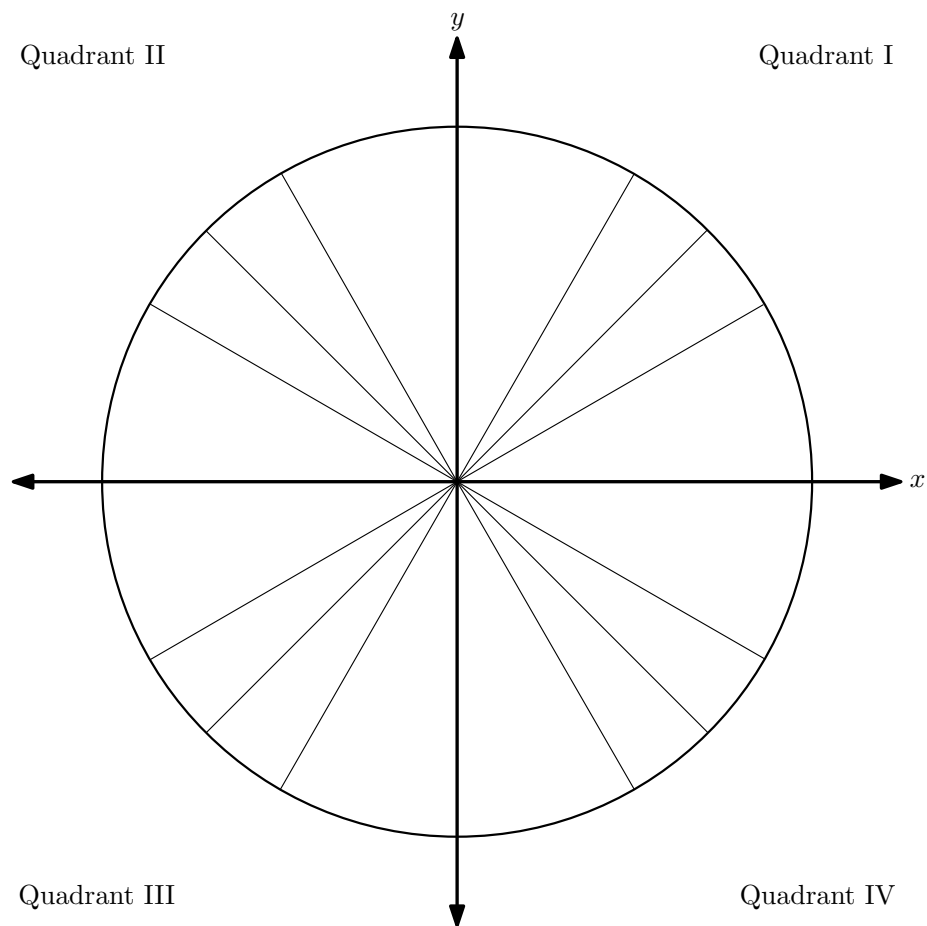


## Blank Unit Circle

The unit circle is defined as  $x^2 + y^2 = 1$ , which is a circle with radius 1, centered at the origin  $(0, 0)$ . It is used in trigonometry to simplify finding values of trig functions. An empty unit circle is below with markings to be filled in:



## Practice Problems

1. Find the radian value of  $120^\circ$ .
2. Find the coordinates corresponding to the radian value  $\frac{7\pi}{6}$ .
3. Find all angles  $\theta$  between 0 and  $2\pi$  such that  $|\sin \theta| = \frac{\sqrt{3}}{2}$ .
4. In which quadrants is  $\tan \theta$  positive?

## Solutions

1.

$$120^\circ \cdot \frac{\pi}{180^\circ} = \boxed{\frac{2\pi}{3}}$$

2. We wish to find the  $x$ -coordinate  $\cos \frac{7\pi}{6}$  and the  $y$ -coordinate  $\sin \frac{7\pi}{6}$ . First note that

$$\pi < \frac{7\pi}{6} < \frac{3\pi}{2}$$

so the angle lies in quadrant III. In quadrant III both  $x$ - and  $y$ -coordinates are negative. The reference angle of  $\frac{7\pi}{6}$  is

$$\frac{7\pi}{6} - \pi = \frac{\pi}{6}$$

Therefore, the coordinates are

$$\left( \cos \frac{7\pi}{6}, \sin \frac{7\pi}{6} \right) = \left( -\cos \frac{\pi}{6}, -\sin \frac{\pi}{6} \right) = \boxed{\left( -\frac{\sqrt{3}}{2}, -\frac{1}{2} \right)}$$

3. If

$$|\sin \theta| = \frac{\sqrt{3}}{2}$$

then if  $\theta'$  is the reference angle, then

$$\sin \theta' = \frac{\sqrt{3}}{2}$$

Since  $\theta'$  is less than  $\frac{\pi}{2}$ , we can use the 30 – 60 – 90 right triangle to see that

$$\theta' = \frac{\pi}{3}$$

The value of  $\theta$  in quadrant II is

$$\pi - \theta' = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

The value of  $\theta$  in quadrant III is

$$\pi + \theta' = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

The value of  $\theta$  in quadrant IV is

$$2\pi - \theta' = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

Thus, all the angles are

$$\boxed{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$$

4. Since

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$\tan \theta$  is positive only if both  $\sin \theta$  and  $\cos \theta$  are positive or both  $\sin \theta$  and  $\cos \theta$  are negative.  $\sin \theta$  and  $\cos \theta$  represent  $y$ - and  $x$ -coordinates. Both  $x$ - and  $y$ -coordinates are positive in quadrant I and both are negative in quadrant III. Therefore,  $\tan \theta$  is positive in quadrants I and III. Note that  $\tan \theta$  is 0 if  $\sin \theta = 0$ , which happens on the  $x$ -axis. In addition,  $\tan \theta$  is undefined if  $\cos \theta = 0$ , which happens on the  $y$ -axis.