Trigonometry Solutions

1. (a) $90^\circ = 90^\circ \cdot \dfrac{2\pi}{360^\circ} = \boxed{\dfrac{\pi}{2}}$

   (b) $135^\circ = 135^\circ \cdot \dfrac{2\pi}{360^\circ} = \boxed{\dfrac{3\pi}{4}}$

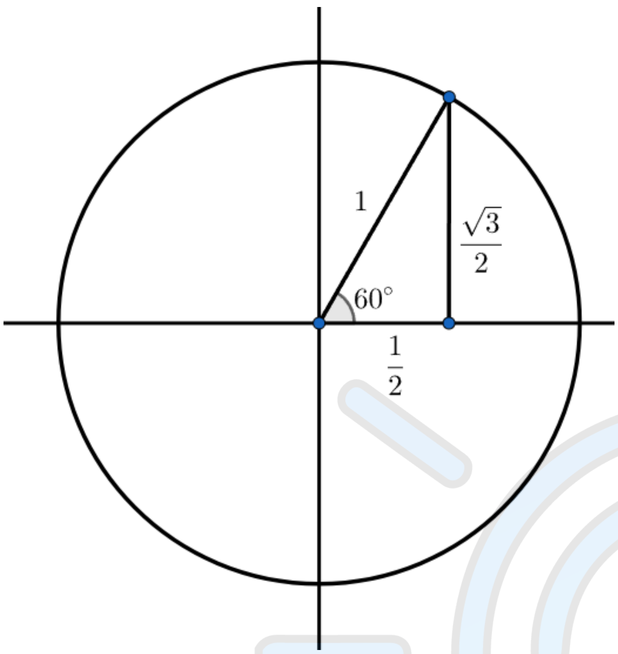
   (c) $405^\circ = 405^\circ \cdot \dfrac{2\pi}{360^\circ} = \boxed{\dfrac{9\pi}{4}}$

2. (a) $\dfrac{11\pi}{4} = \dfrac{11\pi}{4} \cdot \dfrac{360^\circ}{2\pi} = \boxed{495^\circ}$

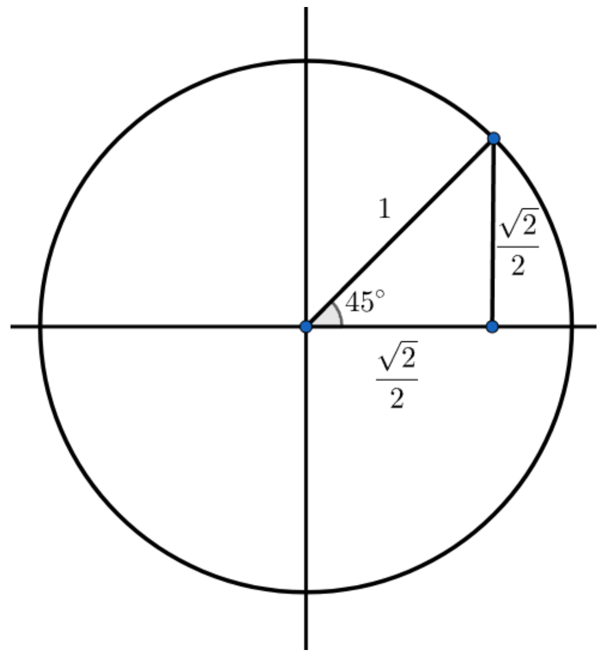
   (b) $\dfrac{7\pi}{6} = \dfrac{7\pi}{6} \cdot \dfrac{360^\circ}{2\pi} = \boxed{210^\circ}$

   (c) $\dfrac{\pi}{2} + \dfrac{\pi}{3} = \dfrac{5\pi}{6} = \dfrac{5\pi}{6} \cdot \dfrac{360^\circ}{2\pi} = \boxed{150^\circ}$

3. (a) Using the fact that 30-60-90 right triangles have side ratios of $\dfrac{1}{2} : \dfrac{\sqrt{3}}{2} : 1$, we obtain that $\sin 60^\circ = \boxed{\dfrac{\sqrt{3}}{2}}$.

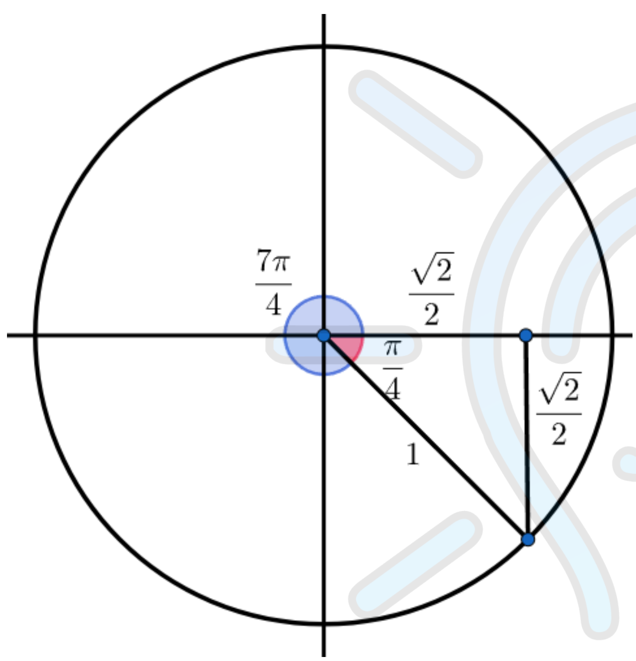


(b) Using the fact that 45-45-90 right triangles have side ratios of $1 : 1 : \sqrt{2}$, we obtain that $\cos 45^\circ = -\frac{\sqrt{2}}{2}$. Since 135° is in quadrant 2 with reference angle 45°, $\cos 135^\circ = -\cos 45^\circ = \boxed{-\frac{\sqrt{2}}{2}}$.



(c) We observe that $\sec \pi = \frac{1}{\cos \pi} = \frac{1}{-1} = \boxed{-1}$.

(d) Notice that $\sin \frac{\pi}{4} = \cos \frac{\pi}{4}$, so $\tan \frac{\pi}{4} = \frac{\sin \frac{\pi}{4}}{\cos \frac{\pi}{4}} = 1$. Since $\frac{7\pi}{4}$ is in quadrant 4 and the reference angle is $\frac{\pi}{4}$, $\tan \frac{7\pi}{4} = -\tan \frac{\pi}{4} = \boxed{-1}$.



4. (a) By the Pythagorean Theorem, we observe that $8^2 + 15^2 = a^2$. Solving for a, we get that $\boxed{a = 17}$. Similarly, using the Pythagorean Theorem we have $15^2 + 20^2 = b^2$, so $\boxed{b = 25}$.

   (b) Note that $\tan x = \dfrac{AD}{BD} = \dfrac{15}{20} = \boxed{\dfrac{3}{4}}$. Similarly, $\sec y = \dfrac{AC}{CD} = \boxed{\dfrac{17}{8}}$.

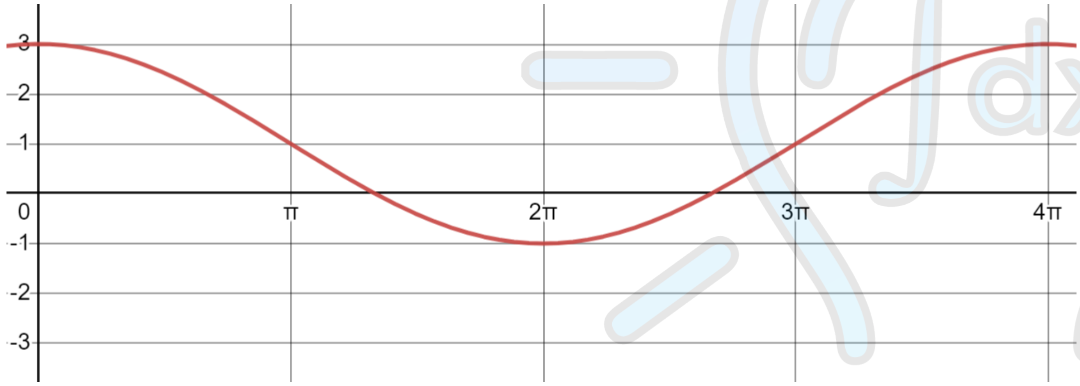
   Lastly, $\sin(x + y) = \sin x \cdot \cos y + \cos x \cdot \sin y = \dfrac{15}{b} \cdot \dfrac{8}{a} + \dfrac{20}{b} \cdot \dfrac{15}{a} = \boxed{\dfrac{84}{85}}$

5. The general form of the equation of a sine function is $f(x) = a \sin(bx - c) + d$, where a is the amplitude, $\dfrac{2\pi}{b}$ is the period, $\dfrac{c}{b}$ is the phase shift, and $y = d$ is the midline. Plugging in the given values, we obtain that $a = 2$, $b = 2$, $c = 0$, and $d = 4$. Thus, our desired equation is $\boxed{f(x) = 2\sin 2x + 4}$. Notice that there are other possible answers, since the sine function is periodic. For example, $f(x) = 2\sin(2x + 6\pi) + 4$ and $f(x) = 2\sin(2x - 12\pi) + 4$ result in the same sine curve.

6. The period of $f(x) = 2\cos\left(\dfrac{x}{2}\right) + 1$ is 4π. We can create a table of values for $0 \leq x \leq 4\pi$ at certain points to help us sketch $f(x)$:

x	$f(x)$
0	3
π	1
2π	-1
3π	1
4π	3

After plotting these points and connecting them by a curve, we obtain the graph



7. We rewrite all trigonometric functions in terms of $\sin x$ and $\cos x$:

$$\frac{(\csc^2 x)(\tan x)}{2\cot x} = \frac{\left(\frac{1}{\sin^2 x}\right)\left(\frac{\sin x}{\cos x}\right)}{2\left(\frac{\cos x}{\sin x}\right)} = \boxed{\frac{1}{2\cos^2 x}}$$

8. (a) We observe that the equation is equivalent to

$$2\sin x = \frac{\sin x}{\cos x} \Rightarrow 2\sin x - (\sin x)\frac{1}{\cos x} = 0 \Rightarrow \sin x\left(2 - \frac{1}{\cos x}\right) = 0.$$

It follows that either $\sin x = 0$, or $\cos x = \frac{1}{2}$. Thus, the solutions are $\boxed{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}}$.

(b) Notice that $\sin^2 x + \cos^2 x = 1$. The equation $\sin^2 x + 2\cos^2 x = 2$ can be rewritten as $\sin^2 x + \cos^2 x + \cos^2 x = 2$, which simplifies to $\cos^2 x = 1$. It follows that either $\cos x = 1$ or $\cos x = -1$, and our solutions are $\boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$.

9. We will use the Pythagorean Identity $\sin^2 x + \cos^2 x = 1$ twice. This gives us that

$$\sin x = \sqrt{1 - \cos^2 x} = \frac{1}{2}.$$

We select the positive root since $\sin x$ is positive because x is in quadrant 1. Secondly, since $\tan y = \sqrt{3}$, $\frac{\sin y}{\cos y} = \sqrt{3} \Rightarrow \sin y = \sqrt{3} \cdot \cos y \Rightarrow \sin^2 y = 3\cos^2 y$, so by the Pythagorean Identity

$$\sin^2 y + \cos^2 y = 1 \Rightarrow 3\cos^2 y + \cos^2 y = 1 \Rightarrow \cos y = \frac{1}{2}.$$

This means that $\sin y = \frac{\sqrt{3}}{2}$. Again, we select the positive solutions since y is also in quadrant 1.

Finally, we see that $\cos(x + y) = \cos x \cos y - \sin x \sin y = \frac{\sqrt{3}}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \boxed{0}$.

10. Using the fact that $1 + \tan^2 x = \sec^2 x$ and the fact that $\tan x$ is negative in quadrant 4,

$$1 + \tan^2 x = \sec^2 x \Rightarrow \tan^2 x = \sec^2 x - 1 = 3 \Rightarrow \tan x = \boxed{-\sqrt{3}}.$$

11. We want to find an equation that describes the graph in the form $f(x) = a\sin(bx - c) + d$. Note that the graph of the function decreases when x increases from 0, which means that a is negative (if a was positive, the function would be increasing). Since the amplitude of the graph is $\frac{1}{2}$, we know that $a = -\frac{1}{2}$. Next, we note that the period of the function is π, so $b = \frac{2\pi}{\pi} = 2$. Since the graph doesn't have a phase shift, we obtain that $c = 0$. Lastly, the midline is $y = 1$, so $d = 1$.
Thus, the graph of the function is $\boxed{f(x) = -\frac{1}{2}\sin x + 1}$. Notice that there may be other possible solutions because $\sin x$ is a periodic function, such as $-\frac{1}{2}\sin(x + 2\pi) + 1$.

12. (a) $\sin x$ is defined for all real values of x, so the domain of $f(x)$ is $\boxed{x \in \mathbb{R}}$. Since the range of $\sin x$ is $[-1, 1]$ (and the same for $\sin 2x$), we obtain that the lowest value of $f(x)$ is $-2 + 5 = 3$, and the highest value is $2 + 5 = 7$. Thus, the range of $f(x)$ is $\boxed{y \in [3, 7]}$.

    (b) $\cos x$ is defined for all real values of x, so the domain of $g(x)$ is $\boxed{x \in \mathbb{R}}$. Since the range of $\cos x$ is $[-1, 1]$, we obtain that the lowest value of $g(x)$ is $-7 - 2 = -9$, and the highest value is $7 - 2 = 5$. Thus, the range of $g(x)$ is $\boxed{y \in [-9, 5]}$.

    (c) The domain of $\arcsin(2x)$ is the same as the range of $\sin(2x)$ because $\arcsin x$ is the inverse of $\sin x$ restricted to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Hence, the domain of $h(x)$ is $\boxed{x \in [-1, 1]}$ and the range is $\boxed{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$.

13. (a) Observe that

$$\arcsin\left(\frac{AB}{AC}\right) = \frac{\pi}{3}$$
$$m\angle C = \frac{\pi}{3}$$
$$\beta = \frac{\pi}{2} - m\angle C = \boxed{\frac{\pi}{6}}.$$

    (b) Since $\tan\beta = \frac{4}{3}$, we may let $AB = 3, BC = 4$ to find the ratio $\frac{BC}{AC}$. By the Pythagorean Theorem, ΔABC is a 3-4-5 right triangle, so $AC = 5$ and $\arcsin\left(\frac{BC}{AC}\right) = \arcsin\left(\frac{4}{5}\right) \approx \boxed{53.1^\circ}$

14. Note that

$$\tan 20^\circ = \frac{AB}{2000+x}$$
$$\tan 50^\circ = \frac{AB}{x}.$$

From this, it is clear that $x \cdot \tan 50^\circ = (2000+x) \cdot \tan 20^\circ$, which is a linear equation in one variable. We solve it to obtain

$$x(\tan 50^\circ - \tan 20^\circ) = 2000 \cdot \tan 20^\circ \Rightarrow x = \frac{2000 \cdot \tan 20^\circ}{\tan 50^\circ - \tan 20^\circ} \approx \boxed{879.4 \text{ ft}}$$

15. (a) Since $\sin \beta = \frac{BC}{AC} = \frac{5}{18}$, we see that $BC = \frac{5}{18} AC = \boxed{\frac{5}{2}}$.

(b) Because $\tan \beta = \frac{15}{8}$, to find the ratio $\frac{AB}{AC}$, we may let $BC = 15$ and $AB = 8$. Then $AC = \sqrt{AB^2 + BC^2} = \sqrt{8^2 + 15^2} = 17$, so $\frac{AB}{AC} = \boxed{\frac{8}{17}}$.

(c) Note that $\cos(90^\circ - \beta) = \sin \beta$. We have that

$$AC = \sqrt{3^2 + 5^2} = \sqrt{34}$$

Therefore, it follows that $\cos(90^\circ - \beta) = \frac{BC}{AC} = \frac{3}{\sqrt{34}} = \boxed{\frac{3\sqrt{34}}{34}}$.