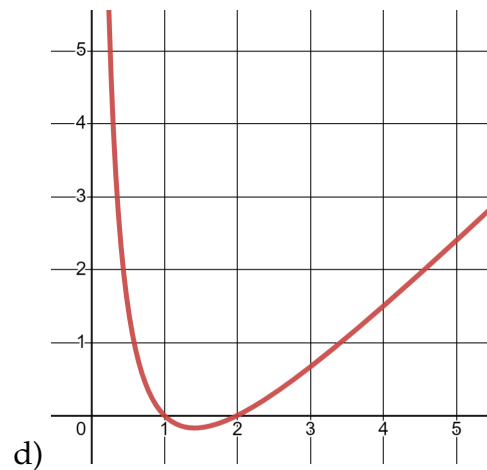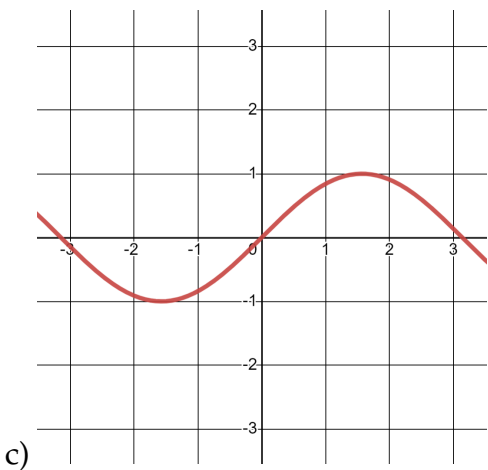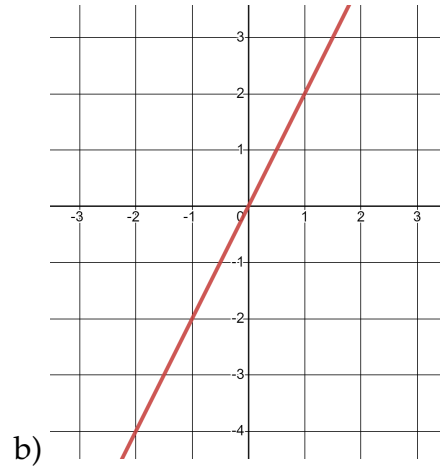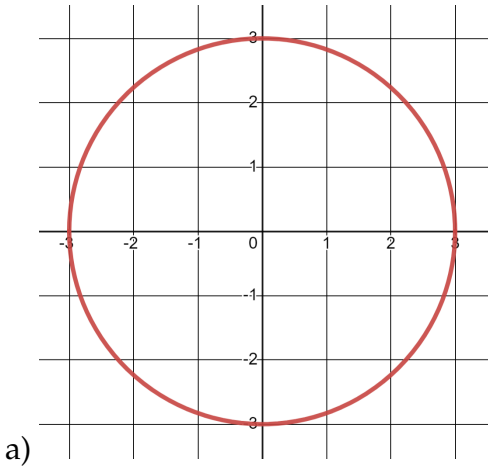# Functions Problems

1. For each of the graphs below, determine whether they represent a function.

a)

b)

c)

d)

2. Express the domain of each of the functions below in interval notation:

(a) $f(x) = \sqrt{x-3}$

(b) $g(x) = \dfrac{1}{x^2 - 9}$

(c) $h(x) = \dfrac{2}{1 - \sqrt{x-9}}$

3. Express the range of each of the functions below in interval notation:

   (a) $f(x) = -(x-1)^2 + 4$

   (b) $f(x) = \sqrt{x+1} + 3$

   (c) $f(x) = 3|x+5| + 2$

4. Given that $f(x) = -3x - 10$ and $g(x) = x^2$, find $f^{-1}(g(-3))$.

5. Find $f(f(3))$ given that

$$f(x) = \begin{cases} 3x - 7 & \text{if } x \geq 3 \\ x^2 - 5 & \text{if } x < 3 \end{cases}$$

6. Sketch the graph of $k(x)$ given that

$$k(x) = \begin{cases} |x| & \text{if } x > 5 \\ \dfrac{x+1}{2} & \text{if } 3 \leq x < 5 \\ x - 5 & \text{if } x < 3 \end{cases}$$

7. For each polynomial, determine whether it increases or decreases when x approaches positive infinity or negative infinity:

   (a) $f(x) = -x^3 + 3x^2 + 2x + 1$

   (b) $f(x) = x^4 + 2x^3 + 1$

   (c) $f(x) = -x^8 + x^5 + x^3 + 1$

   (d) $f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$

8. Determine the interval on which the graph of the function $f(x) = (x-5)^2 - 4$ is negative.

9. Determine the intervals on which the graph of the function $f(x) = (x-1)(x^2 - 4)$ is positive. (Hint: Factor $f(x)$.)

10. Find the equation of each line, in a specified form, given the following information:

    (a) a line with slope $m = 5$ that passes through $(3,7)$; point-slope form.

    (b) a line perpendicular to the line with slope $\dfrac{1}{2}$ that passes through $(0, 2)$; slope-intercept form.
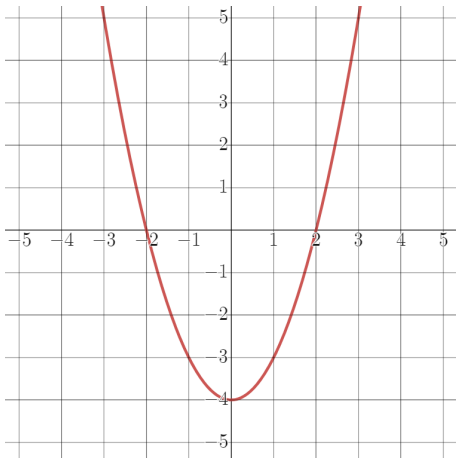
    (c) a line parallel to the line with slope -3 that passes through $(4, 4)$; point-slope form.

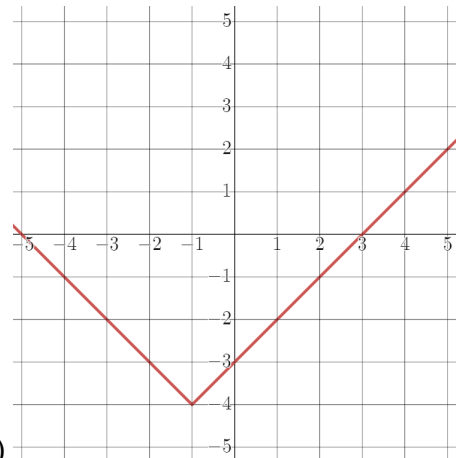11. Find the intersection of the lines ℓ and m:

    line ℓ passes through the points $(3, 5)$ and $(4, 9)$.

    line m passes through the point $(39, 13)$ and has the slope $\dfrac{1}{3}$.
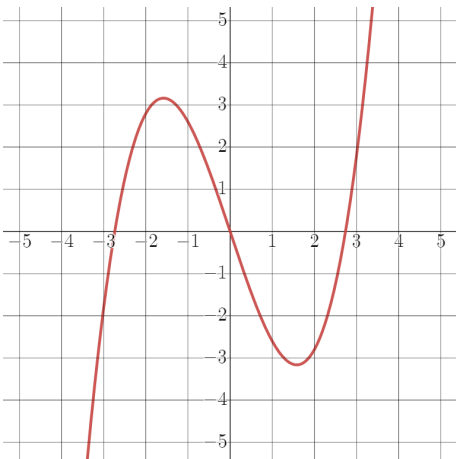
12. For each of the graphs below, determine whether they are even, odd, or neither. Make sure to explain your reasoning.
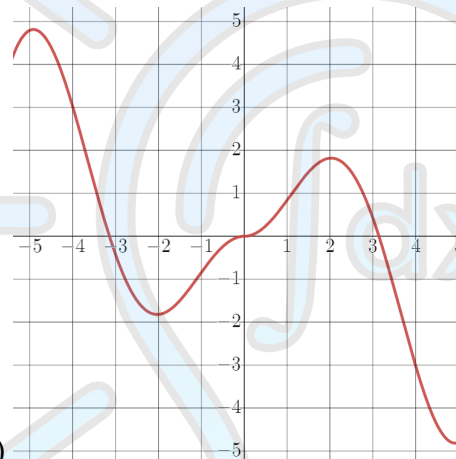
a)

b)

c)

d)

13. Find the inverse of each of the following functions:

    (a) $f(x) = 2x + 5$

    (b) $h(x) = \sqrt{x+1}$

    (c) $q(x) = \dfrac{4x - 7}{2x + 1}$

14. Find the equation of the linear function $f(x)$ given that $f^{-1}(3) = 5$ and $f^{-1}(9) = -7$.

15. Find the x- and y-intercepts of the following functions:

    (a) $f(x) = x^2 - 6x + 9$

    (b) $f(x) = x^3 - 5x^2 - 4x + 20$

    (c) $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$