Functions Solutions

1. In a function for each input value (x-value), there is only one output value (y- value). We can check whether a graph represents a function by performing the **vertical line test**. If every vertical line intersects the graph in at most one point, then the graph represents a function. If some vertical line intersects the graph in more than one point, then the graph does not represent a function.

   a) This is a graph of a circle, and it does not pass the vertical line test. Therefore, it is $\boxed{\text{not a function}}$.

   b) This is a graph of a line, and it passes the vertical line test. Therefore, it is $\boxed{\text{a function}}$

   c) This graph passes the vertical line test. Therefore, it is $\boxed{\text{a function}}$.

   d) This graph passes the vertical line test. Therefore, it is $\boxed{\text{a function}}$.

2. (a) For this function to be defined, the expression inside the square root must be non-negative, as otherwise, the square root would return a complex number. Thus, we have that $x - 3 \geq 0 \Rightarrow x \geq 3$. The domain is then $\boxed{[3, \infty)}$.

   (b) $g(x)$ is only undefined when the denominator is equal to 0. Therefore, we must solve the equation

$$x^2 - 9 = 0$$
$$(x-3)(x+3) = 0$$
$$x = 3, -3.$$

   Therefore the domain of $g(x)$ contains all real numbers except -3 and 3 and in interval notation it is $\boxed{(-\infty, -3) \cup (-3, 3) \cup (3, \infty)}$.

   (c) We first notice that the expression inside the square root must be non-negative. Thus, we have that $(x - 9) \geq 0 \Rightarrow x \geq 9$. However, we also must exclude the values that make the denominator zero, namely $\sqrt{x-9} = 1 \Rightarrow x \neq 10$. Therefore, the domain of $h(x)$ is $\boxed{[9, 10) \cup (10, \infty)}$.

3. We can find the range of each function by considering the set of all possible outputs and the shape of the graph of the function.

   (a) Since $(x-1)^2$ takes on all values greater than or equal to 0, $-(x-1)^2+4$ takes on all values less than or equal to 4. Hence, the range of $f(x) = -(x-1)^2+4$ is $\boxed{(-\infty, 4]}$.

   (b) Since $\sqrt{x+1}$ takes on all values greater than or equal to 0, $\sqrt{x+1}+3$ takes on all values greater than or equal to 3. Hence, the range of $f(x) = \sqrt{x+1}+3$ is $\boxed{[3, \infty)}$.

   (c) Since $|x+5|$ takes on all values greater than or equal to 0, $3|x+5|+2$ takes on all values greater than or equal to 2. Hence, the range of $f(x) = 3|x+5|+2$ is $\boxed{[2, \infty)}$.

4. We first note that $g(-3)$ is $(-3)^2 = 9$, so we want to find $f^{-1}(9)$. That is, we want to find the value x such that $f(x) = 9$. This is equivalent to solving the equation

$$
\begin{aligned}
-3x - 10 &= 9 \\
-3x &= 19 \\
x &= -\frac{19}{3}.
\end{aligned}
$$

   Thus, $f^{-1}(9) = \boxed{-\frac{19}{3}}$. Note that we didn't have to find $f^{-1}(x)$, although we could do it.

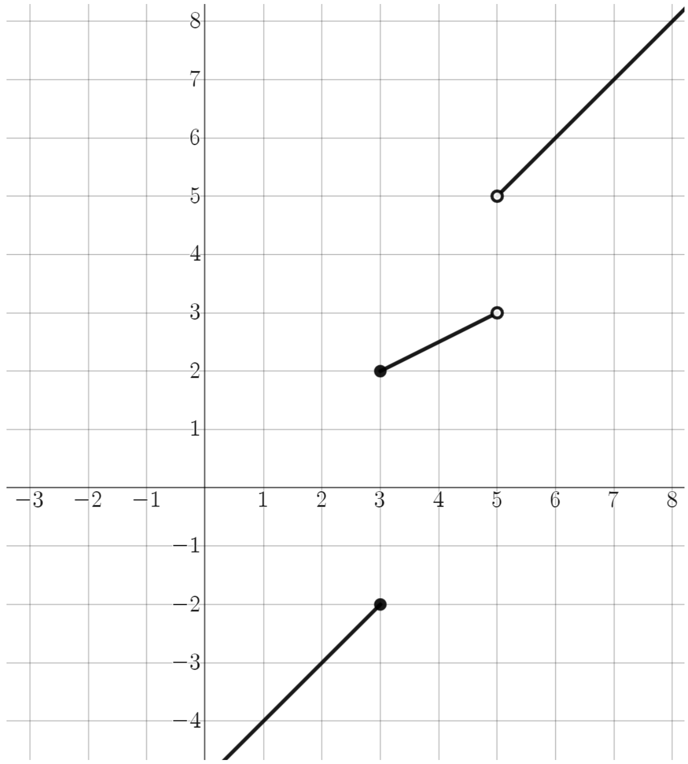
5. We first evaluate $f(3)$. Since $f(x) = 3x - 7$ for $x \geq 3$,

$$f(3) = 3(3) - 7 = 2.$$

   We obtain that $f(f(3)) = f(2)$. Since $f(x) = x^2 - 5$ for $x < 3$,

$$f(2) = 2^2 - 5 = \boxed{-1}.$$

6. $k(x)$ is a piece-wise function, so we need to sketch each section of $k(x)$ separately:



7. (a) To determine the end behavior of $f(x)$, we examine the leading term (term with the greatest degree). In this case, the term is $-x^3$. As x approaches ∞, $-x^3$ approaches $-\infty$. As x approaches $-\infty$, $-x^3$ approaches ∞. Therefore, as x approaches ∞, $f(x)$ approaches $-\infty$. As x approaches $-\infty$, $f(x)$ approaches ∞.

   (b) The leading term is x^4. Because the exponent is even, x^4 will approach ∞ when x approaches either ∞ or $-\infty$. That is, as x approaches ∞, $f(x)$ approaches ∞ and as x approaches $-\infty$, $f(x)$ approaches ∞.

   (c) The leading term is $-x^8$. Because the exponent is even, $-x^8$ will approach $-\infty$ when x approaches either ∞ or $-\infty$. That is, as x approaches ∞, $f(x)$ approaches $-\infty$ and as x approaches $-\infty$, $f(x)$ approaches $-\infty$.

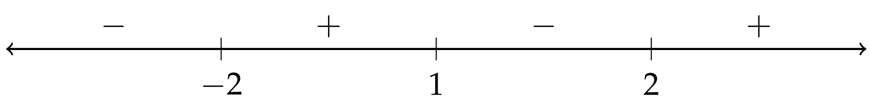
   (d) The leading term is x^5. As x approaches ∞, x^5 approaches ∞. As x approaches $-\infty$, x^5 approaches $-\infty$. Therefore, as x approaches ∞, $f(x)$ approaches ∞. As x approaches $-\infty$, $f(x)$ approaches $-\infty$.

8. Note that $f(x)$ is only negative when $(x-5)^2 < 4$. We now solve for x the inequality

$$\begin{aligned} &(x-5)^2 < 4 \\ &-2 < x-5 < 2 \\ &3 < x < 7. \end{aligned}$$

Therefore, the interval in which $f(x)$ is negative is $\boxed{(3,7)}$.

9. Note that we can factor $f(x)$ using difference of squares to obtain $(x-1)(x-2)(x+2)$. Since $f(x)$ changes sign only at $x = -2, 1, 2$, we can create a sign chart to analyze the sign of $f(x)$ between those values. We can select a test value from each interval to determine the sign of $f(x)$ in that interval. For example, $f(0) = 4 > 0$, so $f(x)$ must be positive in the interval $(-2, 1)$.



From the sign chart, it is clear that $f(x)$ is positive only on $\boxed{(-2,1) \cup (2,+\infty)}$.

10. (a) The point-slope form of the line must be $\boxed{y - 7 = 5(x-3)}$.

(b) Since a perpendicular line has slope $\frac{1}{2}$, we know that the line that we are looking for has slope -2. Since it passes through $(0,2)$, it must have the y-intercept 2. The slope-intercept form of the line must be $\boxed{y = -2x + 2}$.

(c) Since a parallel line has slope -3, the line that we are looking for must also have slope -3. The point-slope form is, therefore, $\boxed{y - 4 = -3(x-4)}$

11. The first line has the slope $\frac{9-5}{4-3} = 4$. Using the point-slope form, we can solve the equa-

tion:

$$\begin{aligned} y - 5 &= 4(x-3) \\ y - 5 &= 4x - 12 \\ y &= 4x - 7. \end{aligned}$$

The equation of the other line can also be found using the point-slope form:

$$\begin{aligned} y - 13 &= \frac{1}{3}(x-39) \\ y - 13 &= \frac{1}{3}x - 13 \\ y &= \frac{1}{3}x. \end{aligned}$$

To find the intersection of the two lines, we solve

$$4x - 7 = \frac{1}{3}x \Rightarrow \frac{11x}{3} = 7 \Rightarrow x = \frac{21}{11}$$
$$y = \frac{1}{3}\left(\frac{21}{11}\right) = \frac{7}{11}.$$

Therefore, the intersection point is $\boxed{\left(\frac{21}{11}, \frac{7}{11}\right)}$.

12. A graph is even if it is symmetric about the y-axis and odd if it is symmetric about the origin.

(a) This graph is symmetric about the y-axis, but not about the origin, so it is $\boxed{\text{even}}$ and not odd.

(b) This graph is not symmetric about the y-axis nor the origin so it is $\boxed{\text{neither}}$ even nor odd.

(c) This graph is symmetric about the origin, but is not symmetric about the y-axis. Therefore, it is $\boxed{\text{odd}}$ and not even.

(d) This graph is symmetric about the origin, but is not symmetric about the y-axis. Therefore, it is $\boxed{\text{odd}}$ and not even.

13. To find the inverse function, we switch the input, x, and the output, y, and then solve for y.

(a)

$$x = 2y + 5$$
$$x - 5 = 2y$$
$$\boxed{y = \frac{x-5}{2}}.$$

(b)

$$x = \sqrt{y+1}$$
$$x^2 = y + 1$$
$$\boxed{y = x^2 - 1}, x \geq 0.$$

We need to add the condition $x \geq 0$, since in the first line x must be non-negative.

(c)

$$x = \frac{4y-7}{2y+1}$$
$$2xy + x = 4y - 7$$
$$2xy - 4y = -x - 7$$
$$y(2x - 4) = -x - 7$$
$$\boxed{y = \frac{x+7}{4-2x}}.$$

14. We may rewrite the information in the problem as

$$f^{-1}(3) = 5 \Rightarrow f(5) = 3$$
$$f^{-1}(9) = -7 \Rightarrow f(-7) = 9.$$

In other words, the line must pass through the points $(5, 3)$ and $(-7, 9)$. The slope of the line is $\dfrac{9-3}{-7-5} = \dfrac{6}{-12} = -\dfrac{1}{2}$. We now use the point-slope form to solve for y:

$$\begin{aligned} y-3 &= -\frac{1}{2}(x-5) \\ y-3 &= -\frac{1}{2}x + \frac{5}{2} \\ y &= -\frac{1}{2}x + \frac{11}{2} \\ &\boxed{f(x) = -\frac{1}{2}x + \frac{11}{2}}. \end{aligned}$$

15. To find the x-intercepts of a function, we set y to 0 and solve for x. To find the y-intercept of a function, we set x to 0 and solve for y.

(a) $f(x) = x^2 - 6x + 9$ can be factored as $(x-3)^2$. First, we find the x-intercepts:

$$\begin{aligned} (x-3)^2 &= 0 \\ x &= \boxed{3}. \end{aligned}$$

To find the y-intercepts, we set x to 0 in the original expression, which is $f(0) = \boxed{9}$.

(b) We can factor $f(x)$ using grouping.

$$(x^3 - 5x^2) - (4x - 20) = (x^2 - 4)(x-5) = (x+2)(x-2)(x-5).$$

The x-intercepts are the roots of $f(x)$, which are $\boxed{2, -2, 5}$. The y-intercept is $f(0) = \boxed{20}$.

(c) Note that the coefficients of $f(x)$ form a row of the Pascal's Triangle. The polynomial can therefore be factored as $(x-1)^4$. Solving $(x-1)^4 = 0$ yields one intercept $x = \boxed{1}$. The y-intercept is $f(0) = \boxed{1}$.