Exponents and Logarithms Problems

1. Given that $\log a = 5$, $\log b = 2$, and $\log c = 1$, find the value of $\log \left(\dfrac{a^3 b^4}{c^7} \right)$.

2. Simplify the following logarithmic expressions:

   (a) $\log_2 80 - \log_2 5$

   (b) $2^{\log_2 17}$

   (c) $\log_2 5 \cdot \log_5 8$

3. Solve for the value of x in the following equations:

   (a) $3 \cdot 7^x + 1 = 5$

   (b) $5e^x + 1 = \dfrac{2e^x + 3}{2}$

   (c) $4^x - 3 \cdot 2^{x+1} + 5 = 0$ (Hint: This is a quadratic in 2^x.)

4. Solve each equation:

   (a) $\dfrac{\log_2 y}{\log_2 5} = 3$

   (b) $\log_2 y + \log_2 (y - 4) = 2$

   (c) $\log_2 (3y) = \log_4 19$

5. Find all values of the variable that satisfy the following inequalities:

   (a) $\log_3 (4a - 7) \leq 3$

   (b) $\log_7 (5x - 4) \geq \log_7 (2x + 8)$

6. Find the x-intercepts and the y-intercept of the graph $y = 5 \cdot 3^{2x} - 15$.

7. Find the domain and the range of the function $f(x) = \log_3 (3x - 7)$.

8. Determine whether the following functions are increasing or decreasing:

   (a) $h(x) = \log_5 (2x + 1) - 17$

(b) $g(x) = \log_7(-2x + 15) + 5$

(c) $f(x) = 3^{-x} - 2$

9. Find the inverse of the function $k(x) = \dfrac{1}{e^x + 1}$.

10. Find the inverse of the function $p(x) = \log_7(2x + 1)^5$.

11. Graph the following functions:

(a) $f(x) = 3^{x-1}$

(b) $q(x) = 2^x - 2$

(c) $h(x) = \log_2(x + 1) - 3$

12. Fully expand the following logarithms:

(a) $\ln(a^2 b^3)$

(b) $\log(abc)^{\frac{5}{2}}$

(c) $\log\left(100\sqrt[3]{a}b^4 / \sqrt{c^5}\right)$

13. The half-life of caffeine in the bloodstream is about 5 hours. For instance, if someone has ingested 200 mg of caffeine, then in 5 hours the amount of caffeine would have reduced to 100 mg. If Manuel ingests 200 mg of caffeine, how long will it take for the amount of caffeine in his body to reduce to 20 mg? Round your answer to three decimal places.

14. James wants to have at least $1,000,000$ in his bank account before he retires. He currently has $800,000$ dollars in stock growing continuously at a rate of 10% every year. How long will it take for him to reach his goal? Round your answer to three decimal places.

15. A population of 1500 rabbits doubles every year. How long will it take for this population to reach $1,000,000$? Assume that the population increases continuously. Round your answer to three decimal places.