Algebra Basics Solutions

1. We can simplify the following expression by using PEMDAS. The operation that is being performed is underlined:

$$\begin{aligned} &5 + 3 \cdot \underline{(4^2 - 2)} \div 7 \cdot 2 \\ &= 5 + \underline{3 \cdot (14)} \div 7 \cdot 2 \\ &= 5 + \underline{42 \div 7} \cdot 2 \\ &= 5 + \underline{6 \cdot 2} \\ &= \underline{5 + 12} \\ &= \boxed{17}. \end{aligned}$$

2. Since the ratio of boys to girls is $3 : 2$, let x be the number of groups of 3 boys and 2 girls in the math class. Thus, there are $3x$ boys and $2x$ girls. In total, there are $3x + 2x = 5x$ children. It is given that $5x = 25$, so $x = 5$ and there are $2x = \boxed{10}$ girls.

3. We start by simplifying the equation using the distributive property. Then we combine the like terms and isolate the variable to solve for x. The operation that is being performed is underlined:

$$\begin{aligned} \underline{5(3x - 7)} + 1 &= 4x - 1 \\ 15x \underline{- 35 + 1} &= 4x - 1 \\ 15x - 34 &= 4x - 1 \\ 15x - 4x &= -1 + 34 \\ 11x &= 33 \\ x &= \boxed{3}. \end{aligned}$$

4. Using the first equation, we can solve for the value of y in terms of x:

$$\begin{aligned} 3x + y &= 6 \\ y &= 6 - 3x. \end{aligned}$$

We can substitute this expression into the second equation to solve for x:

$$\begin{aligned} -2x + 3y &= -15 \\ -2x + 3(6 - 3x) &= -15 \\ -2x + 18 - 9x &= -15 \\ -11x &= -33 \\ x &= \boxed{3}. \end{aligned}$$

Lastly, we substitute $x = 3$ into the expression we got for y:

$$\begin{aligned} y &= 6 - 3x \\ y &= 6 - 3(3) \\ y &= -3. \end{aligned}$$

The solution is $\boxed{(x, y) = (3, -3)}$.

5. We can solve this system by the elimination method. Let's multiply the second equation by 4 and subtract it from the first equation:

$$\begin{cases} 8x + 4y &= 9 \\ 2x + y &= 13 \end{cases} \Rightarrow \begin{cases} 8x + 4y &= 9 \\ 8x + 4y &= 52 \end{cases} \Rightarrow 0 = -43.$$

This is a false statement. Therefore, the system of equations is inconsistent and there are $\boxed{\text{no solutions}}$.

6. Notice that in the equation $x^4 - 13x^2 + 36 = 0$ we have only degree 4, degree 2 and degree 0 terms. We can rewrite the equation as $(x^2)^2 - 13(x^2) + 36 = 0$ and factor it as a trinomial with x^2 as the variable. To do so, we wish to find two numbers that add up to -13 and multiply to 36. We can guess that these numbers are -9 and -4:

$$\begin{aligned} x^4 - 13x^2 + 36 &= 0 \\ (x^2 - 9)(x^2 - 4) &= 0 \\ (x-3)(x+3)(x-2)(x+2) &= 0. \end{aligned}$$

The values of x that make the equation equal to 0 are $\boxed{-3, -2, 2, 3}$.

7. Since the variables are in the exponents, we will simplify constant terms and get the equation where both sides are powers of 2. We can then set the exponents equal to each other and solve for x. Laws of exponents yield

$$\begin{aligned} 4(2^{x+2}) &= \frac{3(2^{3x-5})}{6} \\ 2^2 \cdot 2^{x+2} &= \frac{2^{3x-5}}{2} \\ 2^{x+4} &= 2^{3x-6} \\ x + 4 &= 3x - 6 \\ -2x &= -10 \\ x &= \boxed{5}. \end{aligned}$$

8. We cross multiply the fractions and solve the resulting linear equation:

$$\frac{3x-5}{2x+7} = \frac{5}{3}$$
$$3(3x-5) = 5(2x+7)$$
$$9x-15 = 10x+35$$
$$x = \boxed{-50}.$$

We can mentally check that -50 does not make any of the denominators equal to 0 in the original equation, so it is the answer.

9. We use FOIL to expand the equation and then combine the like terms:

$$\begin{aligned}(3x+5)(2x+7) &= (3x)(2x)+(3x)(7)+(5)(2x)+(5)(7)\\ &= 6x^2+21x+10x+35\\ &= \boxed{6x^2+31x+35}.\end{aligned}$$

10. The area of the rectangle is $x(x-1) = x^2 - x$ and the area of the right triangle is $\frac{5\cdot 8}{2} = 20$. Thus,

$$x^2 - x = 20$$
$$x^2 - x - 20 = 0$$

We guess two numbers that add up to -1 and multiply to -20. They are -5 and 4.

$$(x-5)(x+4) = 0.$$

Therefore, either $x-5=0$ or $x+4=0$, so $x=5$ or $x=-4$. However, side lengths cannot be negative, so we reject $x=-4$. The only solution is $x = \boxed{5}$.

11. To solve for x, we first bring all the terms with x to the same side of the equation, factor out

x and then divide both sides by the coefficient of x.

(a)

$$\frac{x+1}{y} = 5$$

$$x + 1 = 5y$$

$$\boxed{x = 5y - 1}.$$

(b)

$$xy = x + 1$$

$$xy - x = 1$$

$$x(y-1) = 1$$

$$\boxed{x = \frac{1}{y-1}}.$$

(c) We cross multiply the fractions and then solve for x in the resulting linear equation:

$$\frac{2}{3x-y} = \frac{4}{5x}$$

$$10x = 12x - 4y$$

$$-2x = -4y$$

$$\boxed{x = 2y}.$$

12. Following the hint, first perform the multiplication and then factor by grouping:

$$\begin{aligned}(x+1)^2(x-6)+10x+10&=0\\(x^2+2x+1)(x-6)+10x+10&=0\\(x^3-4x^2-11x-6)+10x+10&=0\\x^3-4x^2-x+4&=0\\x^2(x-4)-(x-4)&=0\\(x^2-1)(x-4)&=0\\(x-1)(x+1)(x-4)&=0.\end{aligned}$$

Thus, $x = \boxed{-1,1,4}$.

13. Let the number of apples Ana bought be a. Then the number of bananas she bought is $2a$. The total cost can be expressed as

$$\begin{aligned}0.20a+0.20(2a)&=12.60\\0.20(3a)&=12.60\\a&=21.\end{aligned}$$

Therefore, the number of bananas Ana bought is $2a = \boxed{42}$.

14. $\boxed{\textbf{True}}$.

$$\begin{aligned}2^x\cdot 4^2&=2^x\cdot(2^2)^2\\&=2^x\cdot 2^4\\&=2^{x+4}.\end{aligned}$$

15. **False**. For example, let

$$A = 4$$

$$B = 5$$

$$C = 2B = 10.$$

Then $A : C = 4 : 10 \neq 2 : 1$, so the statement must be false.